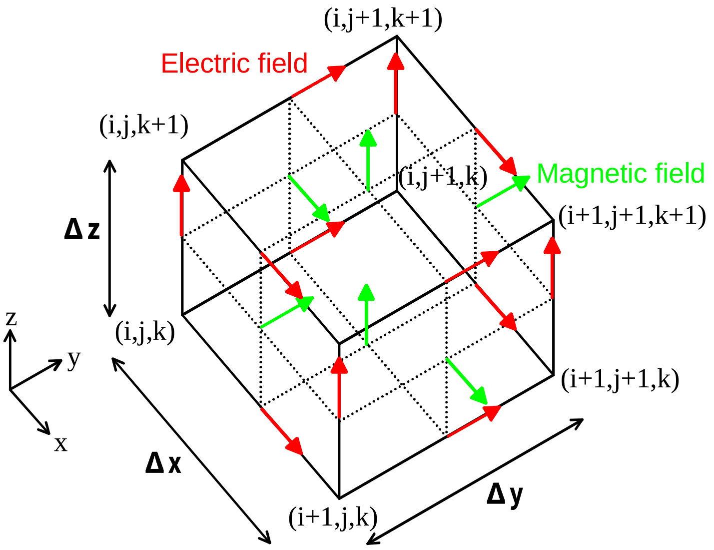
FDTD法
Finite Difference Time Domain method
時間領域差分法



Ando & Hirokawa lab.

Takuichi Hirano (RA)

June 5, 2002

Tokyo Institute of Technology

History

1966

K. S. Yee, “Numerical Solution of Initial Boundary Value Problems Involving Maxwell’s Equations in Isotropic Media,” IEEE Trans. Antennas Propagat., vol. 14, no. 4, pp.302-207, 1966

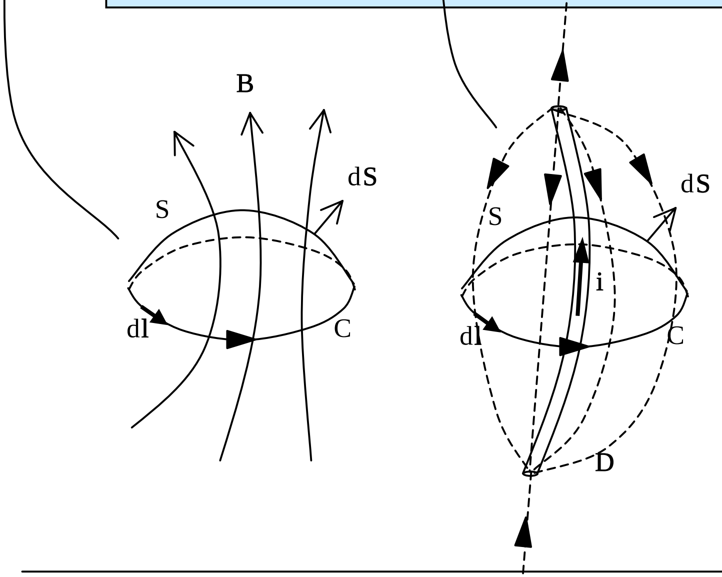
写真提供: http://www.boeing.com/

Maxwell's Equation

Maxwell's Equation:

$$\nabla \times \mathbf{E} + j\omega\mu\mathbf{H} = 0$$
$$\nabla \times \mathbf{H} + j\omega\varepsilon\mathbf{E} = \mathbf{i}$$

$$\nabla \times \mathbf{E} + j\omega\mu\mathbf{H} = 0$$
$$\nabla \times \mathbf{H} + j\omega\varepsilon\mathbf{E} = \sigma\mathbf{E}$$



Discrete Time

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu}\nabla\times\mathbf{E}$$

$$\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\varepsilon}\nabla\times\mathbf{H} - \frac{\sigma}{\varepsilon}\mathbf{E}$$

$$\frac{\mathbf{H}^{n+\frac{1}{2}} - \mathbf{H}^{n-\frac{1}{2}}}{\Delta t} = -\frac{1}{\mu}\nabla\times\mathbf{E}^{n}$$

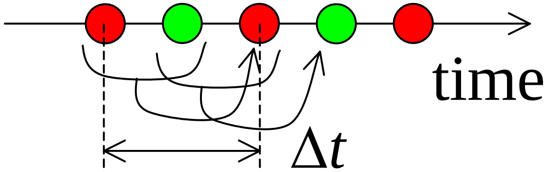
$$\frac{\mathbf{E}^{n} - \mathbf{E}^{n-1}}{\Delta t} = -\frac{\sigma}{\varepsilon}\mathbf{E}^{n-\frac{1}{2}} + \frac{1}{\varepsilon}\nabla\times\mathbf{H}^{n-\frac{1}{2}}$$

$$\frac{\mathbf{E}^{n} + \mathbf{E}^{n-1}}{2}$$

$$\mathbf{E}^{n} = \frac{\boxed{1-\dfrac{\sigma\Delta t}{2\varepsilon}}}{\boxed{1+\dfrac{\sigma\Delta t}{2\varepsilon}}}\mathbf{E}^{n-1} + \boxed{\frac{\Delta t/\varepsilon}{1+\dfrac{\sigma\Delta t}{2\varepsilon}}}\nabla\times\mathbf{H}^{n-\frac{1}{2}}$$

$$\mathbf{H}^{n+\frac{1}{2}} = \mathbf{H}^{n-\frac{1}{2}} - \boxed{\frac{\Delta t}{\mu}}\nabla\times\mathbf{E}^{n}$$

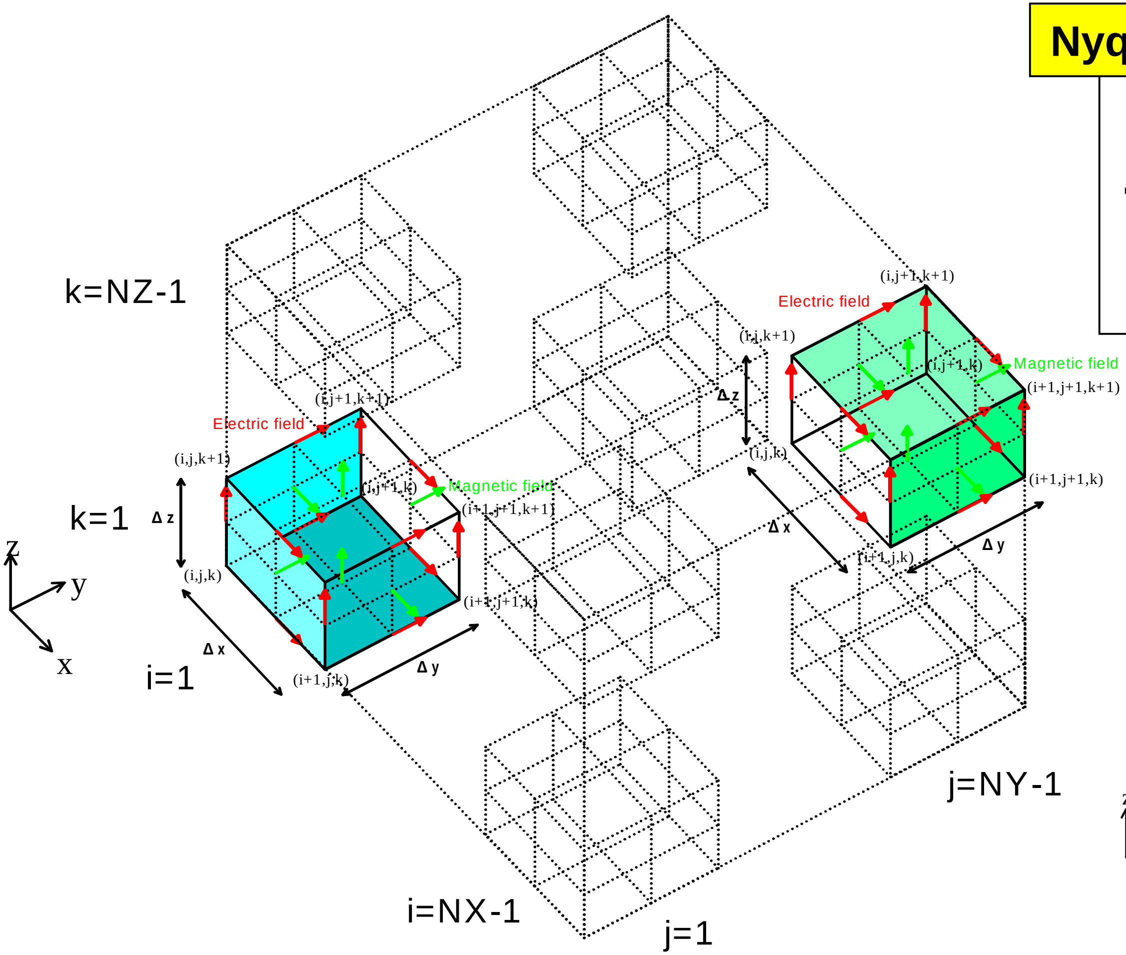
$$\mathbf{E}^{n} = \mathbf{E}(x, y, z, n\Delta t)$$



Courant's stability condition

$$v\Delta t \le 1/\sqrt{(1/\Delta x)^2 + (1/\Delta y)^2 + (1/\Delta z)^2}$$

Yee Cell

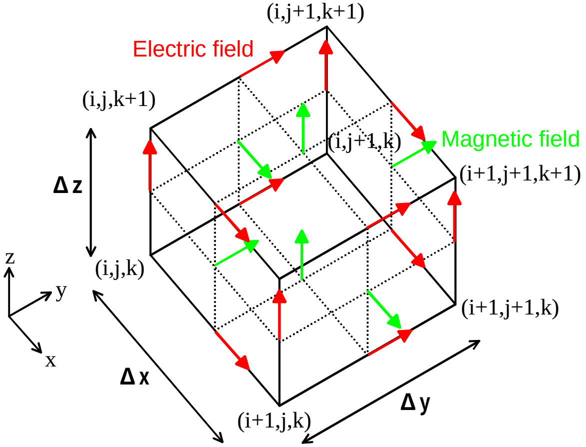


Nyquist's sampling theorem

$$\Delta x, \Delta y, \Delta z < \lambda / 2$$

普通

$$\Delta x = \Delta y = \Delta z \cong \lambda / 10$$



Leap-flog Algorithm

$$\longrightarrow \mathbf{E}^{n-1} \longrightarrow \mathbf{H}^{n-\frac{1}{2}} \longrightarrow \mathbf{E}^{n} \longrightarrow \mathbf{H}^{n+\frac{1}{2}} \longrightarrow$$

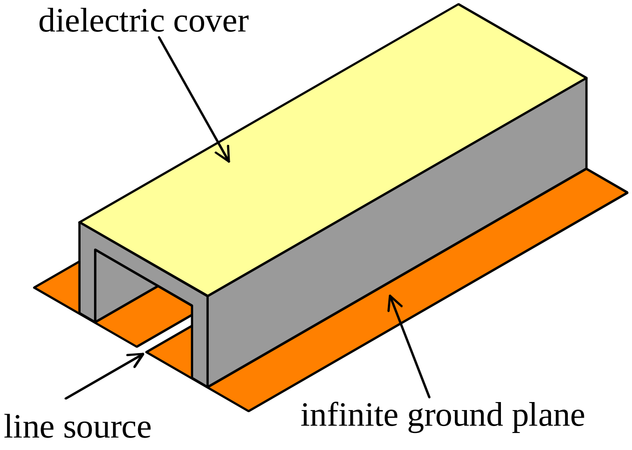
蛙飛び(Leap-flog)**アルゴリズム**

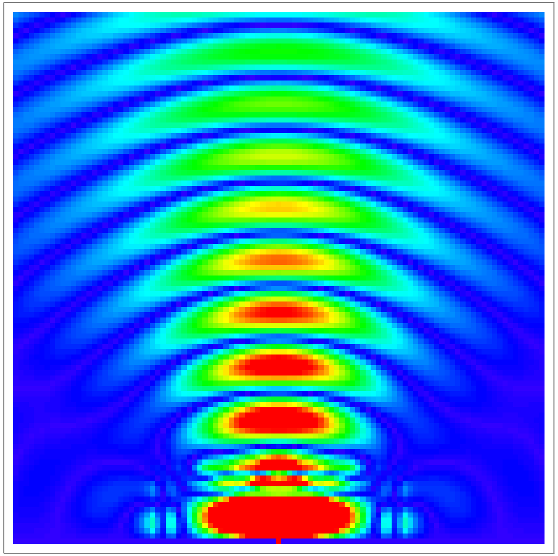
E(x,y,z)
H(x,y,z)

過去の時間の値を保持しなくてもよい

電磁界の時間、場所をずらすことでメモリを節約

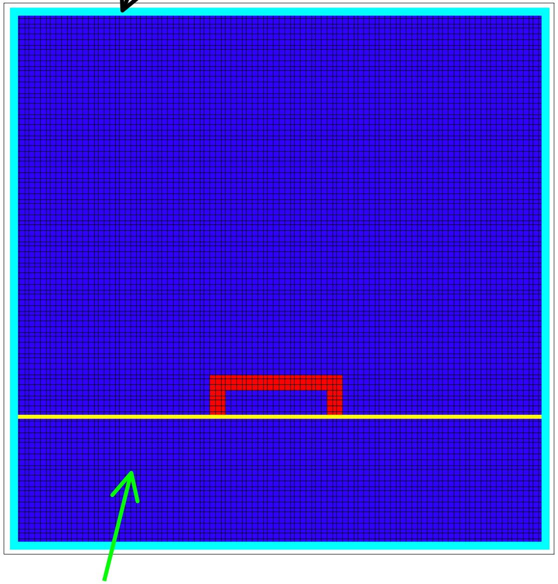
Modeling





Absorbing Boundary Condition (ABC)

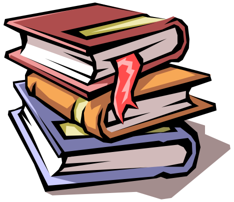
吸収境界条件
(Mur, PML **など**)



一般媒質：更新係数を変えるだけ
PEC: **電界の接線成分を0にする**
PMC: **磁界の接線成分を0にする**

References

[1] 宇野亨、「FDTD法による電磁界およびアンテナ解析」、コロナ社



Comparison

	MoM	FEM	FDTD
速度	○	△	×
メモリ	○	△	×
定式化とプログラム	×	△	○
汎用性	×	○	△
過渡現象	×	△	○

Fine