GSM Solver
– Technical Notes –

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1. Introduction

This document describes algorithm, technology and implementation of GSM Solver.

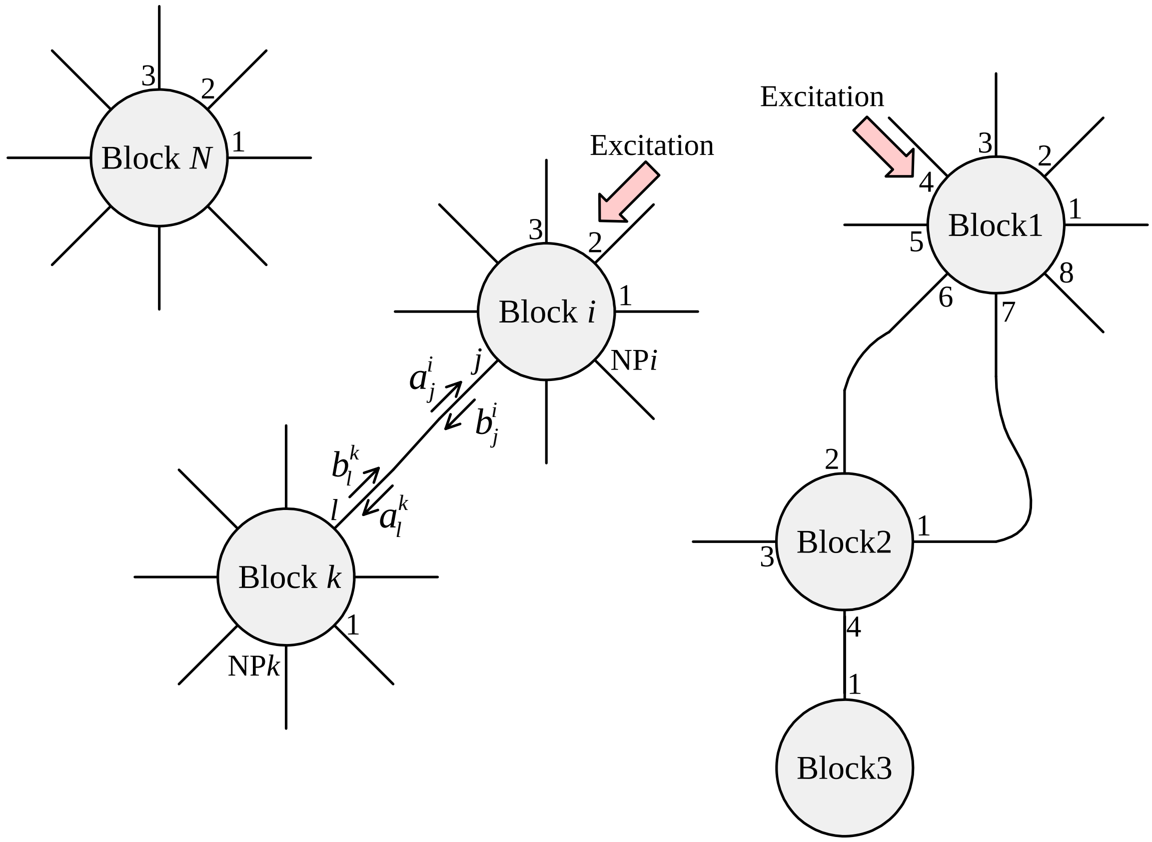

Fig. 1 Arbitrarily-connected scattering matrix network.

2. Program Overview

The program works as follows.

1. Read input files.
2. Make conversion tables, which are needed in the step of making matrix equation.
3. Build matrix equation. (The matrix is square)
4. Solve the matrix equation. (LAPACK routine is used here)
5. Write output files.

3. Tables Used in this Code

Table	Function
tab_gport_2_blk_lport	Look up block no. and its local port no. from global port no.
tab_blk_lport_2_gport	Look up global port no. from block no. and its local port no.
port_connect_info	Look up the pair global port no., which is the connected global port no. If the port is the excitation or load, -1 and 0 are assigned, respectively.
tab_ex_val_lis	List of global port number(s) for excitation.
tab_out_lis	List of global port number(s) for output.
tab_unknown_no	Look up the unknown no. in the GSM matrix equation from global port no. and input/output information. If the position is known, -1 and 0 are returned for excitation and load (0 input).

With above tables, we can freely convert the information which is needed to build the GSM system matrix.

4. Building the GSM System Matrix Equation

Let us look at Port *j* of Block *i* in Fig. 1. There is excitation or matched load at Port *n* of Block *i*, with the excitation coefficient of c_n^i (known coefficient). $c_n^i = 0$ if Port *n* of Block *i* is matched-loaded.

Normal port

The following equations are satisfied for unknowns of input and output.

$$\begin{cases} a_j^i = b_k^l & \text{(input)} \\ b_j^i = \sum_{n=1}^{NP_i} a_n^i S_{jn}^{(i)} & \text{(output)} \end{cases}$$

$$\Leftrightarrow \begin{cases} a_j^i = b_k^l & \text{(input)} \\ b_j^i = \sum_{n \in \text{Normal Port}} a_n^i S_{jn}^{(i)} + \sum_{n \in \text{Excitation}} c_n^i S_{jn}^{(i)} & \text{(output)} \end{cases}$$

$$\Leftrightarrow \begin{cases} a_j^i - b_k^l = 0 \; \text{①} & \text{(input)} \\ b_j^i - \underbrace{\sum_{n \in \text{Normal Port}} a_n^i S_{jn}^{(i)}}_{②} = \underbrace{\sum_{n \in \text{Excitation}} c_n^i S_{jn}^{(i)}}_{③} & \text{(output)} \end{cases}$$

Excitation or matched-loaded port

Input is known and output is unknown.

$$\begin{cases} ---- & \text{(input)} \\ b_j^i = \sum_{n=1}^{NP_i} a_n^i S_{jn}^{(i)} & \text{(output)} \end{cases}$$

$$\Leftrightarrow \begin{cases} ---- & \text{(input)} \\ b_j^i = \sum_{n \in \text{Normal Port}} a_n^i S_{jn}^{(i)} + \sum_{n \in \text{Excitation}} c_n^i S_{jn}^{(i)} & \text{(output)} \end{cases}$$

$$\Leftrightarrow \begin{cases} ---- & \text{(input)} \\ b_j^i - \underbrace{\sum_{n \in \text{Normal Port}} a_n^i S_{jn}^{(i)}}_{②} = \underbrace{\sum_{n \in \text{Excitation}} c_n^i S_{jn}^{(i)}}_{③} & \text{(output)} \end{cases}$$

In the above procedure, equations with the same number of unknowns are obtained.

Source code is implemented as follows.

```
   ! ******** Build GSM Matrix ********

   write(*,*) '**** BUILD GSM MATRIX ****'
   write(1,*) '**** BUILD GSM MATRIX ****'

   ! Zero Clear
   call matrix_zero_clear(gsm_matrix,n_gsm_matrix_unknown,n_gsm_matrix_unknown,n_gsm_matrix_unknown,n_gsm_matrix_
unknown)
   call vector_zero_clear(gsm_rhs_vec,n_gsm_matrix_unknown,n_gsm_matrix_unknown)

   do i=1,n_gsm_total_port
      ! ---- Equation for Input ----
      j=tab_port_connect_info(i)        ! Connected Global Port No.
      if(j > 0) then
         idx_i=tab_unknown_no(i,1)      ! Unknown No. of Self Port #i
         idx_j=tab_unknown_no(j,2)      ! Unknown No. of Connected Port #j

         !     Eq. for ##i: c_{i}=c{j}                                  ⎫
         ! ⇒ Eq. for ##i: c_{i}-c{j}=0                                  ⎬ ①
         gsm_matrix(idx_i,idx_i)=gsm_matrix(idx_i,idx_i)+1.0d0          ⎪
         gsm_matrix(idx_i,idx_j)=gsm_matrix(idx_i,idx_j)-1.0d0          ⎭
      end if

      ! ---- Equation for Output ----
      idx_i=tab_unknown_no(i,2)          ! Unknown No. of Self Port #i

      block_i=tab_gport_2_blk_lport(i).block       ! Block No. of Port #i
      s_i=tab_gport_2_blk_lport(i).loc_port        ! Local Port No. of Port #i

      !     for ##i: c_{i}=Sum c{j} S_{ij}
      ! ⇒ for ##i: c_{i}-Sum c{j} S_{ij}=0
      gsm_matrix(idx_i,idx_i)=gsm_matrix(idx_i,idx_i)+1.0d0

      do k=1,n_port(block_i)                                                         ⎫
         j=tab_blk_lport_2_gport(block_i,k)        ! Global Port No. #k              ⎪
         idx_j=tab_unknown_no(j,1)                    ! Unknown No. of Connected Port #j
                                                                                     ⎪
         s_j=k                                                                       ⎪
                                                                                     ⎪
         select case (idx_j)                                                         ⎪
            case (0)                                                                 ⎬ ②, ③
               ! No Input from Port #j ③                                             ⎪
            case (-1)                                                                ⎪
               ! Excitation from Port #j                                             ⎪
               gsm_rhs_vec(idx_i)=gsm_rhs_vec(idx_i)+tab_ex_val_lis(j)*s_matrix(block_i,s_i,s_j)
            case default                                                         ③   ⎪
               ! Add to GSM System Matrix                                            ⎪
               gsm_matrix(idx_i,idx_j)=gsm_matrix(idx_i,idx_j)-s_matrix(block_i,s_i,s_j) ②
         end select                                                                  ⎭
      end do
   end do
```

5. Solving the GSM System Matrix Equation

Traditional method, such as Gaussian elimination, can be used for matrix inversion because the matrix is square. But in GSM Solver, pseudo inverse, or generalized inverse, is used for matrix inversion. By using pseudo inverse, the general treatment without any modifications in the topology will be possible even when the S-matrices changes so that the determinant of the matrix becomes zero. An example will be shown later.
In the calculation of pseudo inverse, the **singular value decomposition (SVD)** routine "zgesvd.f plus dependencies" of **LAPACK** [1] is used. In the construction of pseudo inverse, the method in the reference [2] or [3] (pp.33-35) is used.

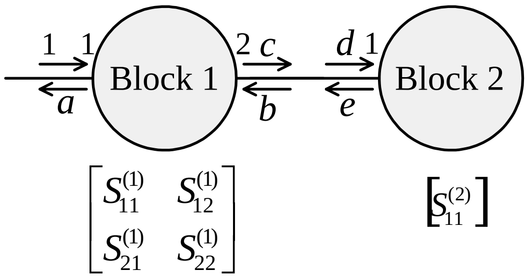

Fig. 2 Example 1

Let us see one of the examples as shown in Fig. 2. The GSM matrix equation can be obtained as follows.

$$\begin{cases} a = S_{11}^{(1)} + bS_{12}^{(1)} \\ b = e \\ c = S_{21}^{(1)} + bS_{22}^{(1)} \\ d = c \\ e = dS_{11}^{(2)} \end{cases} \Leftrightarrow \begin{cases} a - bS_{12}^{(1)} = S_{11}^{(1)} \\ b - e = 0 \\ c - bS_{22}^{(1)} = S_{21}^{(1)} \\ d - c = 0 \\ e - dS_{11}^{(2)} = 0 \end{cases}$$

$$\begin{bmatrix} 1 & -S_{12}^{(1)} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & -S_{22}^{(1)} & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -S_{11}^{(2)} & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} S_{11}^{(1)} \\ 0 \\ S_{21}^{(1)} \\ 0 \\ 0 \end{bmatrix}$$

Here, let us suppose there are no interaction between Port 1 and 2 of Block1 ($S_{12}^{(1)} = S_{21}^{(1)} = 0$).

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & -S_{22}^{(1)} & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -S_{11}^{(2)} & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} S_{11}^{(1)} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Then, the determinant of the above matrix is $1 - S_{11}^{(2)} S_{22}^{(1)}$. When $S_{11}^{(2)} S_{22}^{(1)} = 1$, the matrix becomes singular and the matrix equation becomes indefinite.

In other words, the above equation can be decomposed as follows.

$$a = S_{11}^{(1)}, \begin{bmatrix} 1 & 0 & 0 & -1 \\ -S_{22}^{(1)} & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -S_{11}^{(2)} & 1 \end{bmatrix} \begin{bmatrix} b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

When $S_{11}^{(2)} S_{22}^{(1)} = 1$, the second matrix equation is indefinite. The non-trivial solution of the second matrix equation is the resonant mode of this system. This can be obtained by solving the eigenvalue problem

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ -S_{22}^{(1)} & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -S_{11}^{(2)} & 1 \end{bmatrix} \begin{bmatrix} b \\ c \\ d \\ e \end{bmatrix} = \lambda \begin{bmatrix} b \\ c \\ d \\ e \end{bmatrix},$$

where λ is eigenvalue. The eigenvector $\begin{bmatrix} b & c & d & e \end{bmatrix}^T$ which corresponds to $\lambda = 0$ is the resonant solution. The resonant solution is the non-trivial solution which can exist without any excitations.

But if you just want to know about $a = S_{11}^{(1)}$, the pseudo inverse can be used even in this case. With pseudo inverse, you can solve the problem without any modification in topology input file. Sometimes this is very useful (e.g. in nullification of higher order mode couplings).

References

[1] LAPACK

http://www.netlib.org/lapack/

[2] G. Strang, Linear Algebra and its Applications, 3rd ed., Thomson Learning, 1988.

[3] Numerical Computation – Solution of Linear Equations – (in Japanese)

http://www-antenna.ee.titech.ac.jp/~hira/hobby/edu/em/mom/linear_system/lin_eqs.pdf