ガウス・ルジャンドルの求積法 (Gauss - Legendre Quadrature)

Takuichi Hirano (Tokyo Institute of Technology), November 4, 2008.
Mathematica 6

被積分関数(Integrand)

In[1]:= `f[x_] := Sqrt[1 - x^2]` $f[x\_] := \sqrt{1 - x^2}$

In[2]:= `Integrate[f[x], {x, -1, 1}] // N`

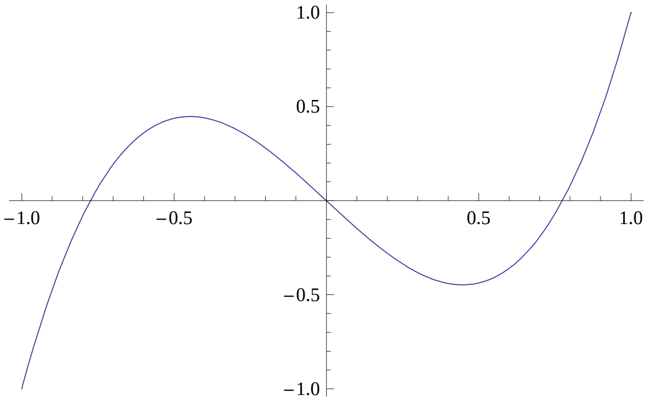
Out[2]= 1.5708

次数を決定

In[3]:= `n = 3;`

In[4]:= `Plot[LegendreP[n, x], {x, -1, 1}]`

Out[4]=



分点の計算

In[5]:= `xlis = x /. Solve[LegendreP[n, x] == 0, x] // N`

Out[5]= {0., −0.774597, 0.774597}

重みの計算

In[6]:= `w[i_] := 1/(D[LegendreP[n, x], x] /. x → xlis[[i]]) * Integrate[LegendreP[n, x]/(x - xlis[[i]]), {x, -1, 1}]`

In[7]:= `Table[w[i], {i, 1, n}] // N`

Out[7]= {0.888889, 0.555556 + 0. ⅈ, 0.555556 + 0. ⅈ}

重みの計算（別解; オリジナルだと思うが、普通に思いつく）

In[8]:=
$$\text{Solve}\left[\text{Table}\left[\sum_{j=0}^{n-1}(\text{ww}[j+1]*\text{LegendreP}[i-1,\ \text{xlis}[[j+1]]])==\text{Integrate}[\text{LegendreP}[i-1,\ x],\ \{x,\ -1,\ 1\}],\ \{i,\ 1,\ n\}\right],\ \text{Table}[\text{ww}[i],\ \{i,\ 1,\ n\}]\right]\ //\ N$$

Out[8]= {{ww[1.] → 0.888889, ww[2.] → 0.555556, ww[3.] → 0.555556}}

ガウス・ルジャンドル積分

In[9]:=
```
a = -1.;
b = 1.;
```
$$\text{xi}[i\_] := \left(\frac{b+a}{2}+\frac{b-a}{2}*\text{xlis}[[i]]\right);$$
$$\frac{b-a}{2}*\sum_{i=1}^{n}w[i]*f[\text{xi}[i]]$$

Out[12]= 1.59162 + 0. i